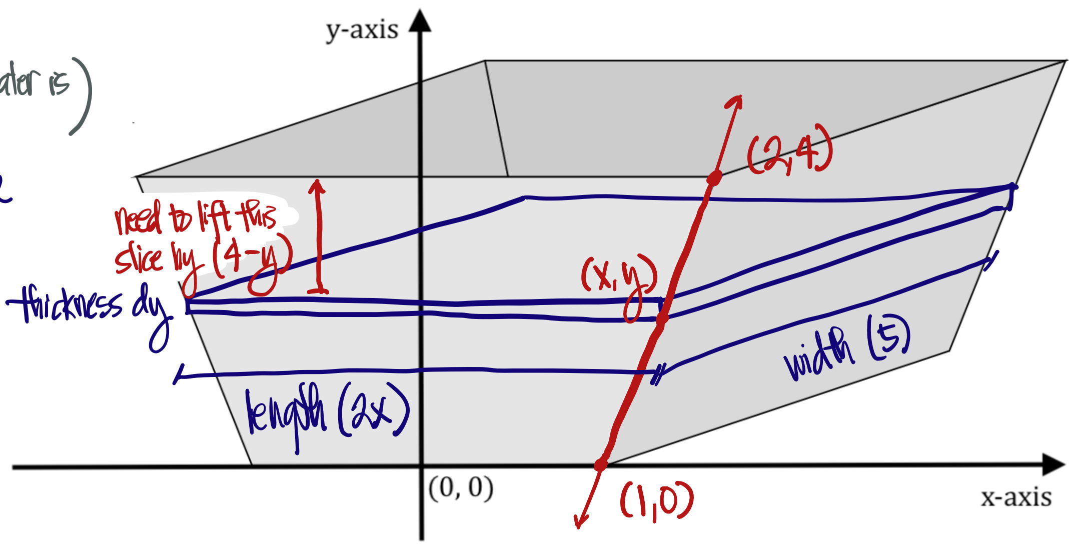
(5) **Pumping - Integrating along the y−axis.** Consider a full watering trough with an isosceles trapazoidal face. The trough is 4 meters tall, 4 meters wide at the top, 2 meters wide at the bottom, and 5 meters long. Set up AND evaluate the integral needed to determine the work required to pump all the water out over the top edge of the trough. Use the weight density of water $9,800 N/m^3$.

Hint: Imagine the bottom of the face of the trough sitting on the x−axis symmetric about (and parallel to) the y−axis. Just like in problem (4), draw a sample slice, label points, identify the needed formula for the boundary and rewrite it as $x = ...$

We slice this solid (geometry-wise since water is a liquid physically) perpendicular to the lifting axis.



★ We model our slices using the line passing through (1,0) and (2,4).

$m = \frac{\Delta y}{\Delta x} = \frac{4-0}{2-1} = 4$; Line: $y - y_0 = m(x - x_0)$; $y - 0 = 4(x-1)$; $y = 4(x-1)$

$\frac{1}{4}y = x - 1$; $x = \frac{1}{4}y + 1 = \frac{1}{4}(y+4)$;

we solve for x since we have dy as the thickness of our slices.

For $y \in [0,4]$: Volume of the slice : $dV = (2x)(5)\,dy = 10\left(\frac{1}{4}\right)(y+4)\,dy$

Force needed to lift slice : $dF = \rho g\, dV = 9800(10)\left(\frac{1}{4}\right)(y+4)\,dy$

Distance the slice needs to lifted by : $4-y$

Work done by lifting the slice : $dW = 9800(10)\left(\frac{1}{4}\right)(4-y)(4+y)\,dy$

$$W = \int_0^4 9800(10)\left(\tfrac{1}{4}\right)(4-y)(4+y)\,dy = 98000\left(\tfrac{1}{4}\right)\int_0^4 16 - y^2\,dy$$

$$= 98000\left(\tfrac{1}{4}\right)\left[16y - \tfrac{1}{3}y^3\right]_0^4 = 98000\left(\tfrac{1}{4}\right)\left[16(4) - \tfrac{1}{3}(4)^3\right] = 98000\left(\tfrac{1}{4}\right)\left(\tfrac{128}{3}\right)$$

$$= \frac{3\,136\,000}{3}\ N\cdot m \approx 1\,045\,333.33\ J \approx 1045\ kJ$$